University of Waterloo

IQC Institute *for* Quantum Computing

Testing contextuality on quantum ensembles with one clean qubit; or: how I learned to stop worrying and love (use) my quantum computer.

O. Moussa, C. A. Ryan, D. G. Cory, R. Laflamme

Institute for Quantum Computing and Dept. of Physics and Astronomy,
University of Waterloo

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1. Contextuality
   - Motivation(s)
   - DIY (a simple proof)
   - BKSC inequality
   - Experimental efforts

2. A bit of quantum computer science
   - circuit for ensemble measurement of correlations

3. Solid-state NMR for QIP
   - primer
   - testing contextuality on ensembles of molecular nuclear spins in the solid state

The main question, in some sense, is

given some preparation, ρ, and some observables

$$A \quad B$$
$$C$$

where

- A and B are compatible;
- A and C are compatible; and
- B and C are not (necessarily) compatible,

does the outcome of observable A, $\nu(A)$, depend on whether one measures B or C simultaneously with A?

Other motivations

- Quantum computers seem to be able to solve some problems more efficiently than their classical counterparts — how come?
- What are small quantum computers good for?

DIY: contextuality

- consider a grid of 3x3 squares that can take on the values ± 1

	c_1	c_2	c_3	$\prod$
r_1	± 1	± 1	± 1	± 1
r_2	± 1	± 1	± 1	± 1
r_3	± 1	± 1	± 1	± 1
$\prod$	± 1	± 1	± 1	

- Evaluate the products of the rows and columns, and let $\alpha = \pi_{r_1} + \pi_{r_2} + \pi_{r_3} + \pi_{c_1} + \pi_{c_2} - \pi_{c_3}$

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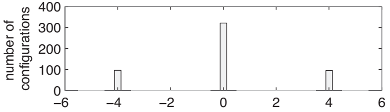

Figure: A histogram of α for all possible 2^9 configurations.

DIY: contextuality – in other words

- for $\alpha = 6$, constrain the products to be:

	c_1	c_2	c_3	$\prod$
r_1	?	?	?	$+1$
r_2	?	?	?	$+1$
r_3	?	?	?	$+1$
$\prod$	$+1$	$+1$	-1	

- are there any satisfying assignments?

Bell[1] -Kochen-Specker[2] -Cabello[3] (BKSC) inequality

(for the Peres[4] -Mermin[5] square)

- Consider a system of 2 qubits prepared according to ρ.
- Measure the correlations between the observables listed in the columns and rows of the following table:

	c_1	c_2	c_3	$\prod$
r_1	$\mathbb{Z}\mathbb{1}$	$\mathbb{1}\mathbb{Z}$	$\mathbb{Z}\mathbb{Z}$	$+\mathbb{1}$
r_2	$\mathbb{1}\mathbb{X}$	$\mathbb{X}\mathbb{1}$	$\mathbb{X}\mathbb{X}$	$+\mathbb{1}$
r_3	$\mathbb{Z}\mathbb{X}$	$\mathbb{X}\mathbb{Z}$	$\mathbb{Y}\mathbb{Y}$	$+\mathbb{1}$
$\prod$	$+\mathbb{1}$	$+\mathbb{1}$	$-\mathbb{1}$	

- Evaluate $\beta = \langle r_1 \rangle + \langle r_2 \rangle + \langle r_3 \rangle + \langle c_1 \rangle + \langle c_2 \rangle - \langle c_3 \rangle$
- where, e.g., $\langle r_1 \rangle = \langle \pi_{\{\mathbb{Z}\mathbb{1},\mathbb{1}\mathbb{Z},\mathbb{Z}\mathbb{Z}\}} \rangle = \langle \mathbb{Z}\mathbb{1} \cdot \mathbb{1}\mathbb{Z} \cdot \mathbb{Z}\mathbb{Z} \rangle_\rho$, and so forth.
- then, for <u>any</u> state ρ:
  - For any NCHV theory: $\beta \leq 4$
  - Quantum Mechanics: $\beta = 6$

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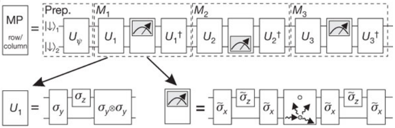
[1] J. S. Bell. *Rev. Mod. Phys.* **38**, 447-452 (1966).
[2] S. Kochen, and E. P. Specker. *J. Math. Mech.* **17**, 59-87 (1967).
[3] A. Cabello, *Phys. Rev. Lett.* **101**, 210401 (2008).
[4] A. Peres. *Physics Letters A.* **151**, 107 (1990).
[5] N. D. Mermin. *Phys. Rev. Lett.* **65**, 3373 (1990).

Experimental efforts

- Two trapped Ca ions [6]

a



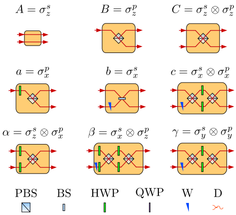
b

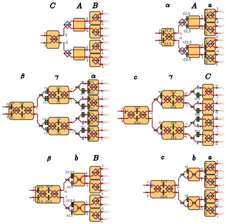
$U[\sigma_x \otimes \sigma_x] = U_Y^{MS}(-\pi/2)\, U_z^{(1)}(\pi/2)$ $\quad U[\sigma_x \otimes \sigma_z] = U_X^{MS}(-\pi/2)\, U_X(\pi/2)$

$U[\sigma_y \otimes \sigma_y] = U_X^{MS}(-\pi/2)\, U_z^{(1)}(\pi/2)$ $\quad U[\sigma_z \otimes \sigma_x] = U_Y^{MS}(-\pi/2)\, U_X(-\pi/2)$

$U[\sigma_z \otimes \sigma_z] = U_Y^{MS}(-\pi/2)\, U_z^{(1)}(\pi/2)\, U_Y(\pi/2)$

- Path and polarization degrees of freedom of a single photon [7]





[6] G. Kirchmair *et al.*, *Nature* **460**, 494 (2009).
[7] E. Amselem *et al.*, *Phys. Rev. Lett.* **103**, 160405 (2009).

Alternatively,

- Introduce a probe two-level system (prepare in $+1$ eigenstate of $\mathbb{X}$)
- To measure any two-outcome observable S:
  - if the system is in a $+1$ eigenstate of S, do nothing; and
  - if it is in a -1 eigenstate, apply a phase flip (pauli-$\mathbb{Z}$) to the probe qubit.
- For $S = P_+ - P_-$, where P_+ and P_- are the projectors on the $+1$ and -1 eigenspaces of S, define

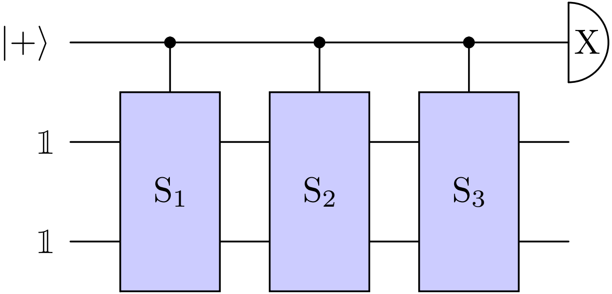
$$U_S = \mathbb{1}_2 \otimes P_+ + \mathbb{Z} \otimes P_-$$

This transformation can also be expressed as a controlled operation dependent on the state of the probe qubit:

$$\begin{aligned} U_S &= \mathbb{1}_2 \otimes P_+ + \mathbb{Z} \otimes P_- \\ &= \tfrac{1}{2}\left(\mathbb{1}_2 + \mathbb{Z}\right) \otimes \left(P_+ + P_-\right) + \tfrac{1}{2}\left(\mathbb{1}_2 - \mathbb{Z}\right) \otimes \left(P_+ - P_-\right) \\ &= |0\rangle\langle 0| \otimes \mathbb{1}_d + |1\rangle\langle 1| \otimes S\,, \end{aligned}$$

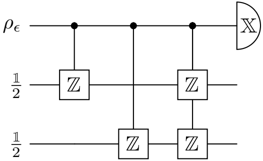
which is unitary for S unitary.

Ensemble measurement of $\langle S_1 S_2 S_3 \rangle_{\mathbb{1}}$

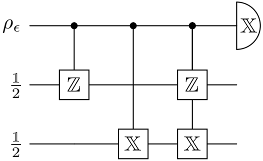


* An ensemble measurement of $\mathbb{X}$ on the probe qubit evaluates to the required correlation.
* Incidentally, this model of computation is known as Deterministic Quantum Computation with one qubit (DQC1)

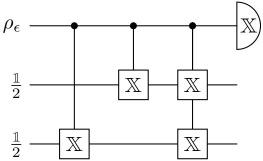
(r_1)



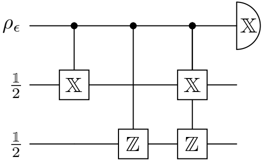
(c_1)



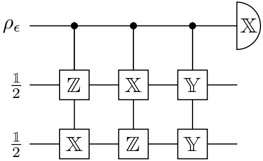
(r_2)



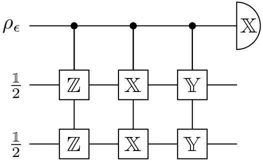
(c_2)

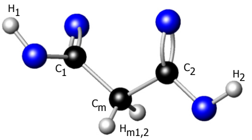


(r_3)



(c_3)





kHz	C_1	C_2	C_m
C_1	6.380	0.297	0.780
C_2	-0.025	-1.533	1.050
C_m	0.071	0.042	-5.650

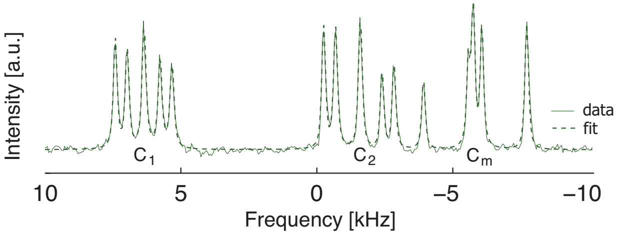


Figure: Malonic acid ($C_3H_4O_4$) molecule and Hamiltonian parameters (all values in kHz). Elements along the diagonal represent chemical shifts, ω_i, with respect to the transmitter frequency (with the Hamiltonian $\sum_i \pi\omega_i \mathbb{Z}_i$). Above the diagonal are dipolar coupling constants ($\sum_{i<j} \pi D_{i,j}(2\ \mathbb{Z}_i\mathbb{Z}_j - \mathbb{X}_i\mathbb{X}_j - \mathbb{Y}_i\mathbb{Y}_j)$, and below the diagonal are J coupling constants, ($\sum_{i<j} \frac{\pi}{2} J_{i,j}(\mathbb{Z}_i\mathbb{Z}_j + \mathbb{X}_i\mathbb{X}_j + \mathbb{Y}_i\mathbb{Y}_j)$. An accurate natural Hamiltonian is necessary for high fidelity control and is obtained from precise spectral fitting.

Experimental results: $\beta = 5.2 \pm 0.1$

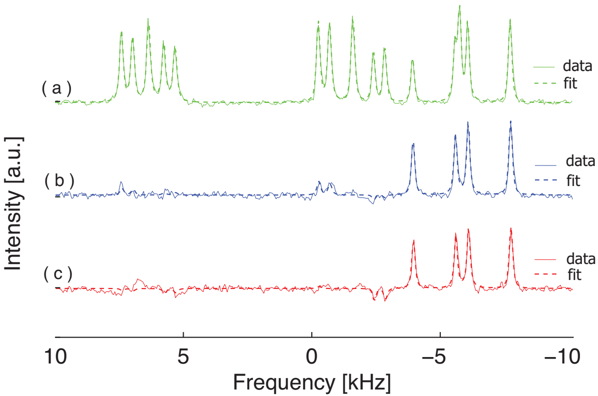


Figure: Summary of experimental results (solid lines) and the corresponding spectral fits (dashed lines). Shown are (a) (in green) a proton-decoupled ^{13}C spectrum following polarization-transfer from the abundant protons; (b) (in blue) the spectrum produced by the initial preparation procedure, $\mathbb{X} \otimes \frac{\mathbb{1}_2}{2} \otimes \frac{\mathbb{1}_2}{2}$; and (c) (in red) the average of the six spectra corresponding to the six terms in β weighted by the appropriate signs.

What I was trying to say is

- BKSC inequality creates a separation between NCHV theories and QM.
  - Non-contextuality is testable ...
  - ... using DQC1
  - ... on ensembles
  - ... of nuclear spins
  - ... in the solid state
  - ... using their magnetic resonance.
- Quantum Information Processing is useful for tasks other than factoring.

If you have been,

Thanks for listening.